

Dihadron fragmentation functions for large invariant mass

J. Zhou and A. Metz

Department of Physics, Temple University, Philadelphia, Pennsylvania 19122-6082, USA

(Dated: January 20, 2011)

Using perturbative Quantum Chromodynamics, we compute dihadron fragmentation functions for a large invariant mass of the dihadron pair. The main focus is on the interference fragmentation function H_1^\triangleleft , which plays an important role in spin physics of the nucleon. Our calculation also reveals that H_1^\triangleleft and the Collins fragmentation function have a closely related underlying dynamics. By considering semi-inclusive deep-inelastic scattering, we further show that collinear factorization in terms of dihadron fragmentation functions, and collinear factorization in terms of single hadron fragmentation functions provide the same result in the region of intermediate invariant mass.

PACS numbers: 12.38.Bx, 12.39.St, 13.85.Ni, 13.87.Fh

I. Introduction.— Fragmentation functions (FFs) for quarks and gluons parameterize the hadronization taking place in high-energy scattering processes with identified strongly interacting particles in the final state. The main focus is typically on FFs describing the transition of a parton into a single hadron — see Ref. [1] for a field-theoretic definition of such objects. However, already in the late 1970's dihadron fragmentation functions (DiFFs) were introduced in order to quantify the hadron structure of jets [2]. Moreover, it was shown that DiFFs are needed to obtain a consistent result for the production of two hadrons in electron-positron annihilation when working beyond leading order in perturbative Quantum Chromodynamics (QCD) [3]. In the meantime, DiFFs also play a considerable role in heavy ion physics — see [4] and references therein.

In 1993, it was proposed [5] that quark fragmentation into two hadrons can also be used to address the transversity distribution h_1 of the nucleon [6, 7]. To this end, one can study the production of two hadrons in semi-inclusive deep-inelastic scattering (DIS) in the current fragmentation region [5]. If the target is transversely polarized, there exists a correlation between the spin vector of the target and the orientation of the plane given by the momenta of the two hadrons. This observable contains the product of h_1 and a new fragmentation function (H_1^\triangleleft in the notation of Ref. [8]), which describes the strength of a correlation between the transverse polarization of the fragmenting quark and the orientation of the hadron plane. Like the transversity, H_1^\triangleleft is chiral-odd, and it results from the interference between two different production amplitudes why it is normally referred to as interference fragmentation function in the literature [9].

Data on the mentioned observable in semi-inclusive DIS have already been obtained by the HERMES and COMPASS Collaborations [10, 11]. The major difficulty is that, *a priori*, both h_1 and H_1^\triangleleft are unknown. Existing models for H_1^\triangleleft [9, 12, 13] are still in a too early stage for getting a quantitative constraint on the transversity. (For a related discussion we refer to [14].) However, as was shown in [15], one can measure two back-to-back hadron

pairs in electron-positron annihilation in order to get a handle on H_1^\triangleleft — see also [16, 17], and Ref. [18] for preliminary data from the Belle Collaboration. A combined analysis of the two processes allows one, in principle, to extract both unknown functions. (First steps towards this goal are outlined in [19].) Such a strategy would be very similar to the combined analysis of the Collins effect [20] in semi-inclusive DIS and in electron-positron annihilation [21, 22], from which first information about the transversity was obtained [22].

The interference FF H_1^\triangleleft and the Collins function (H_1^\perp in the notation of Refs. [23, 24]) can be considered as complementary tools for getting a handle on the transversity distribution, with both having advantages and drawbacks. An important advantage in the case of H_1^\triangleleft is the fact that one can integrate over the total transverse momentum of the two hadrons in the final state, leading to a collinear factorization formula. In contrast, the Collins effect relies on factorization in terms of transverse momentum dependent parton correlators (TMD-factorization) [25–28], which has additional technical complications. On the other hand, when using H_1^\triangleleft , the dependence on the relative transverse momentum of the two hadrons must be kept in order to have a well-defined hadron plane. This implies that H_1^\triangleleft must also depend on the invariant mass M_{hh} of the dihadron system. If M_{hh} is of the order of Λ_{QCD} , DiFFs are entirely non-perturbative objects. In this kinematical region, one can only fit the DiFFs to experimental data or try to estimate them by using some model for the strong interaction in the non-perturbative regime [9, 12, 13]. (As a matter of principle, FFs cannot be computed in lattice gauge theory.)

In the present paper, we apply perturbative QCD in order to evaluate DiFFs for $M_{hh} \gg \Lambda_{QCD}$. The main focus is on the interference FF H_1^\triangleleft . For large M_{hh} , the DiFFs can be expressed as a convolution of hard coefficients and (collinear) single hadron fragmentation correlators. In particular, the calculation determines the behavior of DiFFs as a function of M_{hh} . While the unpolarized DiFF D_1 drops like $1/M_{hh}^2$, the interference DiFF H_1^\triangleleft behaves

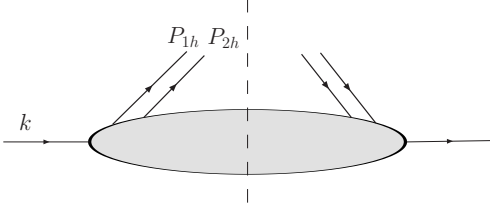


FIG. 1: Graphical representation of dihadron fragmentation function for a quark.

like $1/M_{hh}^3$. We also argue that H_1^\triangleleft and the Collins function H_1^\perp (at large transverse momentum [29]) depend on the same two collinear twist-3 fragmentation correlators, showing that the underlying dynamics of both functions is closely related. In addition, we compute the (transverse) spin dependent cross section for dihadron production in semi-inclusive DIS for $\Lambda_{QCD} \ll M_{hh} \ll Q$ (Q denoting the virtuality of the exchanged photon) with single hadron FFs. By comparing this result with the cross section obtained in the framework of DiFFs, we show that collinear factorization in terms of DiFFs holds as long as $M_{hh} \ll Q$. In its spirit, our study is similar to recent work in which certain transverse momentum dependent parton correlators were evaluated for large transverse momenta, and the matching between collinear factorization and TMD-factorization was explicitly shown for intermediate transverse momenta [29–31] — see also [32] for an overview.

II. Kinematics and definition of dihadron fragmentation functions.— We start by discussing the kinematics for the fragmentation of a quark into two hadrons (displayed for the squared amplitude also in Fig. 1),

$$q(k) \rightarrow h_1(P_{1h}) + h_2(P_{2h}) + X. \quad (1)$$

We assume that the quark has a large light-cone minus momentum k^- . For later convenience, we choose a reference frame in which one hadron has no transverse momentum. In the hadron-1 frame, for instance, the light-cone components of the hadron momenta can be represented as

$$P_{1h} = (0, z_{1h}k^-, 0), \\ P_{2h} = \left(\frac{M_{hh}^2}{2z_{1h}k^-}, z_{2h}k^-, \sqrt{\frac{z_{2h}}{z_{1h}}} M_{hh} \right). \quad (2)$$

Neglecting the hadron masses, one readily verifies that $(P_{1h} + P_{2h})^2 = M_{hh}^2$. We also introduce the total hadron momentum as well as the momentum difference according to

$$P_{hh} = P_{1h} + P_{2h}, \quad R = \frac{P_{1h} - P_{2h}}{2}. \quad (3)$$

Their minus momenta are given by $P_{hh}^- = z_{hh}k^-$, $R^- = \hat{z}_{hh}k^-$, with $z_{hh} = z_{1h} + z_{2h}$ and $\hat{z}_{hh} = (z_{1h} - z_{2h})/2$.

The operator definition of the unpolarized DiFF and the interference DiFF, for a quark flavor q , reads

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{d\hat{z}_{hh}}{32(2\pi)^3} \frac{z_{hh}^2}{z_{1h}^2} \sum_X \int \frac{dy^+}{2\pi} e^{ik^-y^+} \\ \times \langle 0 | \psi^q(y^+) | P_{1h}, P_{2h}, X \rangle \langle P_{1h}, P_{2h}, X | \bar{\psi}^q(0) | 0 \rangle \\ = \frac{\gamma^+}{2} D_1^q(z_{hh}, M_{hh}^2) + \frac{\sigma^{\alpha+} R_{T\alpha}}{2|\vec{R}_T|} H_1^{\triangleleft q}(z_{hh}, M_{hh}^2), \quad (4)$$

where a gauge link has been suppressed. Note that our definition of H_1^\triangleleft differs from the one in Ref. [8] by some prefactors. We have integrated over the relative longitudinal momentum fraction \hat{z}_{hh} , which is needed if one wants to apply the collinear factorization discussed in the next section.

With these conventions, the parton model cross section for the production of two hadrons in semi-inclusive DIS (with a transversely polarized proton), $ep^\uparrow \rightarrow eh_1h_2X$, takes the form

$$\frac{d\sigma}{dx_B dy d\phi_S dz_{hh} dM_{hh}^2 d\phi_R} = \frac{2\alpha_{em}^2 s x_B}{Q^4} \sum_q e_q^2 \\ \times \left[\left(1 - y + \frac{y^2}{2} \right) f_1^q(x_B) D_1^q(z_{hh}, M_{hh}^2) \right. \\ \left. + (1 - y) \sin(\phi_R + \phi_S) h_1^q(x_B) H_1^{\triangleleft q}(z_{hh}, M_{hh}^2) \right]. \quad (5)$$

Equation (5) contains both the unpolarized cross section and one component depending on the transverse target polarization. The azimuthal angle between \vec{R}_T and the lepton plane is denoted by ϕ_R , the azimuthal angle of the transverse spin vector of the proton is denoted by ϕ_S , while x_B and y are the commonly used DIS variables.

III. Dihadron fragmentation functions at large invariant mass.— When $M_{hh} \gg \Lambda_{QCD}$, dihadron fragmentation of a quark can be viewed as a two-step process: first, the quark splits into a quark (with momentum l_q) and a gluon (with momentum l_g), which is calculable in perturbative QCD. Second, each of these two partons fragments into a single hadron. This scenario is illustrated in Fig. 2.

We introduce momentum fractions (z_1, z_2) through

$$l_q = \frac{P_{1h}}{z_1}, \quad l_g = \frac{P_{2h}}{z_2}, \quad (6)$$

and define $\xi = z_{1h}/z_1$. Evaluating the diagram in Fig. 2(a) one finds for the unpolarized DiFF

$$D_1^q(z_{hh}, M_{hh}^2) = \frac{\alpha_s}{2\pi M_{hh}^2} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\hat{z}_{hh} \int_{z_{1h}}^{1-z_{2h}} \frac{d\xi}{\xi(1-\xi)} \\ \times C_F \frac{1+\xi^2}{1-\xi} D_1^{h_1/q}\left(\frac{z_{1h}}{\xi}\right) D_1^{h_2/g}\left(\frac{z_{2h}}{1-\xi}\right), \quad (7)$$

with $D_1^{h_1/q}$ and $D_1^{h_2/g}$ representing unpolarized single-hadron FFs. A second term, where hadron 1 originates

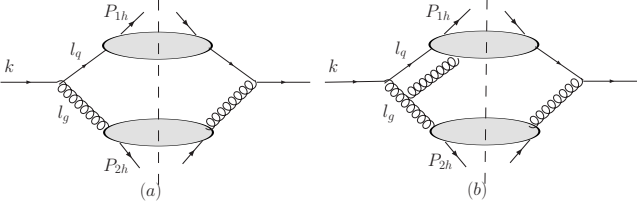


FIG. 2: Generic diagrams contributing to DiFFs for large invariant mass. For the unpolarized DiFF D_1 the diagram (a) needs to be considered, whereas in the case of H_1^Δ both diagrams are relevant.

from the fragmentation of the gluon, is not written for brevity. The result in Eq. (7) shows that the DiFF D_1 drops like $1/M_{hh}^2$. Note also that (7) allows one to recover the inhomogeneous part of the evolution equation for D_1 by integrating over M_{hh}^2 [2, 3].

Computing H_1^Δ for large M_{hh} is much more involved as one has to use collinear twist-3 factorization [33, 34]. One contribution arises from the diagram in Fig. 2(a), if the relative transverse momentum between the quark with momentum l_q and the hadron 1 is kept. Also, 3-parton correlators including a transverse gluon field A_\perp enter the calculation — see the sample diagram in Fig. 2(b). The correlators associated with these two contributions (the so-called ∂_\perp -contribution and the A_\perp -contribution) are of the form $\langle \bar{\psi} \partial_\perp \psi \rangle$ and $\langle \bar{\psi} A_\perp \psi \rangle$, respectively. In this paper, contributions from 3-gluon correlators are neglected since they do not affect any of our general results. We leave this part, details of the calculation, and the discussion about a potential singularity in (7) at $\xi = 0$ and $\xi = 1$ for future work [35].

The calculation of the hard coefficient for the A_\perp -contribution is essentially identical to the corresponding part in the treatment of the Collins function at large transverse momentum [29]. On the other hand, to obtain the ∂_\perp -contribution is more complicated than for the Collins function evaluation. To this end, we assign a (relative) transverse quark momentum according to

$$l_q = \frac{P'_{1h}}{z'_1} + l_{q\perp}, \quad (8)$$

where $z'_1 = z_1 + \delta z_1$, and $P'_{1h} = P_{1h} + \delta P_{1h}$. To keep M_{hh} fixed, P_{2h} must also change, i.e., $P'_{2h} = P_{2h} + \delta P_{2h}$. In particular, $\delta P_{2h\perp} = -z_2 l_{q\perp}$. The kinematics is entirely determined by the constraints

$$\begin{aligned} P_{1h} \cdot \delta P_{1h} &= 0, & P_{2h} \cdot \delta P_{2h} &= 0, \\ P_{1h} \cdot \delta P_{2h} + P_{2h} \cdot \delta P_{1h} &= 0, \\ \delta P_{1h}^- + \delta P_{2h}^- &= 0, & \delta P_{1h\perp} &= 0, \end{aligned} \quad (9)$$

where we use the on-shell condition for the two hadrons, constraints from keeping M_{hh} and z_{hh} fixed, and the fact that we are working in the hadron-1 frame. (Recently, we used a related approach for computing a particular

single spin asymmetry in the Drell-Yan process [36].) The solution to the set of equations in (9) reads

$$\begin{aligned} \delta P_{1h} &= \left(0, \frac{2k^- l_{q\perp}}{P_{2h\perp}} \frac{z_2 z_{1h} z_{2h}}{z_{hh}}, 0 \right), \\ \delta P_{2h} &= \left(-\frac{P_{2h\perp} l_{q\perp}}{k^-} \frac{z_2}{z_{hh}}, -\frac{2k^- l_{q\perp}}{P_{2h\perp}} \frac{z_2 z_{1h} z_{2h}}{z_{hh}}, -z_2 l_{q\perp} \right), \\ \delta z_1 &= \frac{2}{z_{hh}} z_1 z_2 (z_2 - z_1) (1 - \xi) \frac{l_{q\perp}}{P_{2h\perp}}. \end{aligned} \quad (10)$$

The partonic scattering amplitude M depends now on $l_{q\perp}$. We expand M ,

$$\begin{aligned} M(P'_{1h}, P'_{2h}, z'_1) &= M(P_{1h}, P_{2h}, z_1) \\ &+ \frac{\partial M(P'_{1h}, P'_{2h}, z'_1)}{\partial l_{q\perp}} \Big|_{l_{q\perp}=0} l_{q\perp} + \dots \end{aligned} \quad (11)$$

and keep the term linear in $l_{q\perp}$ for obtaining the relevant (twist-3) ∂_\perp -contribution [34, 35].

Both the ∂_\perp -contribution and the A_\perp -contribution can be brought into a gauge invariant form. After collecting all the pieces we find [35]

$$\begin{aligned} H_1^{\Delta q}(z_{hh}, M_{hh}^2) &= \frac{\alpha_s}{2\pi M_{hh}^3} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{d\hat{z}_{hh}}{\sqrt{z_{1h} z_{2h}}} \int_{z_{1h}}^{1-z_{2h}} \frac{d\xi}{\xi} \\ &\times A^{h_1/q}(\hat{z}_{hh}, \xi) D_1^{h_2/g} \left(\frac{z_{2h}}{1-\xi} \right), \end{aligned} \quad (12)$$

where the function A is defined as

$$\begin{aligned} A &= C_F \left[\left(z_1^3 \frac{\partial}{\partial z_1} \frac{\hat{H}(z_1)}{z_1^2} \right) 2\xi^2 \frac{z_1 - z_2}{z_{hh}} + \hat{H}(z_1) \frac{2\xi^2}{1-\xi} \right] \\ &+ \int \frac{d\bar{z}_1}{\bar{z}_1^2} P V \left(\frac{1}{\frac{1}{z_1} - \frac{1}{\bar{z}_1}} \right) \hat{H}_F(z_1, \bar{z}_1) \\ &\times \left[-C_F \frac{2z_{1h}}{z_1} \left(1 + \frac{z_{1h}}{\bar{z}_1} - \frac{z_{1h}}{z_1} \right) \right. \\ &\left. - \frac{C_A}{2} \frac{2z_{1h}}{z_1} \frac{z_1 \bar{z}_1 (z_1 + \bar{z}_1) - z_{1h} (z_1^2 + \bar{z}_1^2)}{z_1 (z_1 - \bar{z}_1) (\bar{z}_1 - z_{1h})} \right]. \end{aligned} \quad (13)$$

This result shows that H_1^Δ behaves like $1/M_{hh}^3$ for large M_{hh} . It also reveals an intimate connection between H_1^Δ and the Collins function (at large transverse momentum) [29], as both functions depend on the same collinear twist-3 correlation functions \hat{H} and \hat{H}_F , which we take in the definition of Ref. [29]. The hard coefficients (of the ∂_\perp -contribution) differ in both cases.

IV. Single spin asymmetry for dihadron production in semi-inclusive DIS—In this section, we investigate the validity of the factorization formula (5) for the cross section of dihadron production in semi-inclusive DIS for the case $\Lambda_{QCD} \ll M_{hh} \ll Q$. Though one might expect this factorization to hold, to the best of our knowledge no explicit supportive calculation exists. We focus on the discussion of the spin-dependent component of the

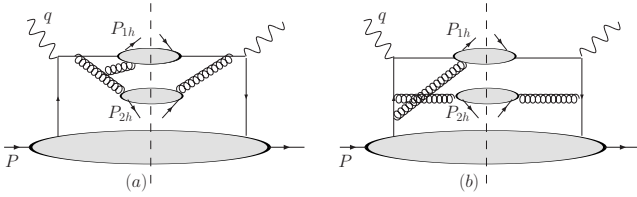


FIG. 3: Sample diagrams contributing to dihadron production in semi-inclusive DIS. Diagram (a) generates a leading power contribution and diagram (b), in the light-cone gauge, is power-suppressed for $M_{hh} \ll Q$.

cross section, which is nontrivial already at lowest order in perturbation theory.

For M_{hh} of the order of Q , one can use collinear factorization in terms of single hadron fragmentation correlators. Sample diagrams are shown in Fig. 3. We have evaluated the cross section for this kinematics, and then expanded the result for $\Lambda_{QCD} \ll M_{hh} \ll Q$. For factorization in terms of DiFFs to hold, the result has to match with (5), if H_1^\perp for $M_{hh} \gg \Lambda_{QCD}$ from (12) is inserted. By inspecting the Feynman diagrams it becomes obvious that this matching is indeed nontrivial. For instance, diagram (b) in Fig. 3 has no counterpart in the calculation of H_1^\perp for large M_{hh} . However, it turns out that for $M_{hh} \ll Q$ this diagram is power-suppressed. This is true only in the light-cone gauge $A^- = 0$ which we use for this analysis. In a covariant gauge, the treatment gets more involved [35]. In the end we indeed find a matching, showing the consistency of the factorization in terms of DiFFs for $M_{hh} \gg \Lambda_{QCD}$ as long as $M_{hh} \ll Q$. For the unpolarized cross section the corresponding analysis is trivial to lowest order, but also becomes nontrivial ones loop corrections are included.

IV. Conclusions.— In this paper, we have studied DiFFs for a large invariant mass M_{hh} of the dihadron pair, where perturbative QCD can be applied. The main focus has been on the interference DiFF H_1^\perp , which drops like $1/M_{hh}^3$ for large M_{hh} and is related to the same universal twist-3 collinear fragmentation correlators that describe the Collins FF H_1^\perp (at large transverse momentum). The analysis also predicts that the transverse single spin asymmetry for dihadron production in semi-inclusive DIS behaves like $1/M_{hh}$. The preliminary COMPASS data [11], ranging up to $M_{hh} \approx 2 \text{ GeV}$, are in agreement with this general result. We expect a corresponding behavior for the so-called Artru-Collins asymmetry in electron-positron annihilation [15], for which preliminary data from Belle exist [18]. For the case of semi-inclusive DIS we have also shown explicitly, to lowest nontrivial order in perturbation theory, that collinear factorization in terms of DiFFs is consistent for large M_{hh} provided that $M_{hh} \ll Q$.

This work is supported by the NSF under Grant No. PHY-0855501.

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